

**2007/11**



Impacts of emission reduction policies in a multi-  
regional multi-sectoral small open economy with  
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CORE DISCUSSION PAPER  
2007/11

**Impacts of emission reduction policies in a multi-regional multi-sectoral  
small open economy with endogenous growth**

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February 2007

**Abstract**

The burden sharing of pollution abatement costs raises the issue of how to share the costs between entities (country, region or industry) and how the pollution permits should be distributed between the parties involved. This paper explores this issue in the framework of a dynamic endogenous growth 2 sectors – 2 regions – 2 inputs Heckscher-Ohlin model of a small open multi-regional economy with an international tradable permits market. Given an "emission-based grand-fathering" sharing rule, capital accumulation is more negatively affected by the environmental policy in the energy intensive sector. We show that such a property does not necessarily hold with a "production-based grand-fathering" sharing rule. We also show that the impact on capital is likely to translate into the sectoral added value level after some time, specially if the economy is submitted to an increasingly constraining environmental policy driving up the ratio price of permits to price of energy. Finally, we show that the impact of environmental policy at the regional level depends crucially on the specialization of the region along the baseline.

**Keywords:** pollution permits, grand-fathering, sectoral spillovers, multi-regional economy, endogenous growth

**JEL classification:** D58, H21, E22, O40

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Raouf Boucekkine acknowledges the support of the French National Agency for Research (ANR) through an ACI contract on sustainable development and technological progress.

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the authors.

# 1 Introduction

The burden sharing of pollution abatement costs, e.g. in the Kyoto Protocol context, raises the issue of how to share the costs between entities (country, region or industry) and how the pollution permits should be distributed between the parties involved in the Protocol. In Belgium, the debate opposes in particular the Flemish and Walloon regions. W.r.t. the other regions, Wallonia is characterized by an industry which is more energy consuming. From the point of view of Flanders, the bulk of the effort should thus be made in Wallonia, where the abatement measures are assumed to be less expensive. On the contrary, this solution is considered to be too unfavourable by Wallonia.

Now it is important to emphasize that the fact that a country's activities are more energy consuming does not necessarily result from their inefficiency. It can also result from the specialization of this country in the production of relatively energy intensive goods, a specialization conditioned itself by its comparative advantages and which should benefit through international trade to all countries involved in it.

In the context of the Belgian burden sharing debate, Germain et al. (2006) develop a model of a 2 sectors - 2 regions small open economy, where each sector produces one good by using fossil energy and another sector specific factor<sup>1</sup>. Sector 1 is more energy intensive and thus more polluting than sector 2. One region is more specialized in the energy intensive sector. Hence, its energy consumption per unit of added value is higher. Given an environmental policy that increases the price of energy (through an energy tax or through the tradable permits price), the authors show as a first result that the energy intensive sector is more burdened than the other sector, so that the region specialized in the energy intensive sector is more burdened than the other region.

Now this intuitive result does not necessarily extend when one takes into account the tax revenues or the tradable permits endowments associated to the control of pollution. The impacts of an emission reduction policy can indeed be modulated by bringing into play such transfers assigned to the regions. In this respect the above authors compute the permits endowments to the regions such that the environmental policy has both the property to be efficient (i.e. abatement marginal costs are equalized between sectors and regions) and fair (i.e. the relative losses of welfare are identical between regions). The endowment is of course relatively more generous for the region specialized in the energy intensive sector. And the difference between regional permits endowments is higher, (i) the higher the national objective in terms of emission reduction, and (ii) the more the regions are unevenly affected by the climate policy.

The analysis of Germain et al. (2006) presents the drawback to be static. However a region or a country's factor endowments and thus its specialization are likely to evolve through time. Consequently the impacts on this region or country of a long term environmental policy (like climate policies) are also likely to evolve through time. On the other hand, if the impacts of an environmental policy on a country's welfare depend on its specialization, the dependence is likely to go the other way around. Indeed it seems probable that an emission reduction policy that translates in an increase of the total cost of energy will induce a change in a country's specialization towards less energy intensive products, and this seems the more likely to happen the more this country is initially specialized in energy intensive goods.

In this paper, we shall consider a dynamic model allowing for specialization rever-

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<sup>1</sup>These specific factors can be considered as aggregates of all non-fossil energy factors (such as capital, labour, infrastructure, non fossil energies,...) with different composition .

sal. One way to get such a property is to incorporate time-dependent spillovers across economic sectors and regions. This might even be the easiest way to generate reversals in specialization. There is an extensive literature about spillovers both at a regional or international level. In particular, the empirical assessment of such spillovers have been at the heart of a quite abundant empirical literature. An important and early contribution to the topic is due to Coe and Helpman (1995) who assessed the economic growth impact of R&D expenditures in OECD countries. They found that such expenditures are beneficial not only for the performing countries but also for the trade partners. Smolny (1999) provided an empirical evaluation of international sectoral spillovers for German and US industries using a broad panel of industry sector data for both countries over the period 1960-1990. In particular, Smolny analyzed productivity convergence, trying to disentangling the precise mechanisms behind. He found that most of the convergence comes from total factor productivity convergence, and more importantly, that endogenous growth models relying on knowledge spillovers are confirmed by the estimates. Lejour and Nahuis (2005) studied in deep the sectoral nature of R&D spillovers and its impact on economic growth. They stressed that the effects of sectoral spillovers do depend on the specialization patterns.

There are also plenty of empirical contributions addressing the issue of spillovers' extent at a regional level. Among them, Van Stel and Nieuwenhuijsen (2004) is a good illustration of this stream of literature. While they did not find any compelling evidence on the growth effect of specialization, in contrast to Lejour and Nahuis (2005), they identified some clear spillovers in certain sectors (specially in the sector services). As one can see, the empirical debate on spillovers is currently much more centered on the kind of relevant spillovers than on the existence of such spillovers, which seems largely admitted. One of the crucial issues turn out to be whether intra-sectoral or inter-sectoral spillovers are more important for economic growth. As to this precise point, the evidence is mixed. A recent study by Malerba, Mancusi and Montobbio (2004) tends however to put forward intra-sectoral spillovers. Using a unique panel data on R&D expenditures and patent citations in 135 narrowly defined technological classes (or sectors) in France, Italy, Japan, United-Kingdom and the US, over the period 1981-1995, they show that the effect of intra-sectoral spillovers is 70% higher than the effect of national inter-sectoral spillovers.

We shall incorporate intra-sectoral spillovers in our model: the industries in a given sector of a given region are assumed to benefit from knowledge spillovers from the industries of the same sector in the other region. As we shall see in Section 2, considering at the same time inter and intra-sectoral spillovers in our model would induce the same long-run capital accumulation in ALL sectors and in ALL regions, which sounds an undesirable outcome as it implies that the sectoral composition of the economy is irrelevant in the long-run. Recent studies on two-sector growth models tend rather to emphasize that investment-specific technological progress (as opposed to the typical Harrod-neutral technological progress at work in the consumption good sector) is likely to generate a persistent productivity gap between the capital good and consumption good sectors, which should translate into different patterns of capital accumulation. This divergence is clearly reflected in the downward trend of the relative price of capital, first pointed out by Gordon (1990), and later exploited in a two-sector accounting framework by Greenwood, Hercowitz and Krusell (1997).

The aim of this paper is to study the impacts of long term environmental policies in the framework of a dynamic 2 sectors - 2 regions - 2 inputs (capital and energy) Heckscher-

Ohlin model of a small open multiregional economy with an international tradable permits market. The main features of the model are the following. Contrary to Germain et al.'s static model, factor endowments are no more exogenous (except in the first period). Sector 1 produces capital goods while sector 2 produces consumer goods. Energy is imported and emissions are proportional to energy use. Sector 1 is more energy intensive than sector 2. The technologies of both sectors are the same in both regions. Because the country is treated as a small open economy, prices are determined by the rest of the world and are thus exogenous. One of the two regions is specialized in the production of the energy intensive good. Growth in endogenous : returns to scale are decreasing at the level of the firm, but because of the technological spillovers, returns to scale are constant at the sectoral level.

The model is an endogenous growth model, where the aggregate productivity of a firm depends on the capital accumulated at the level of the sector and the region to which it belongs. The spillovers are intra-sectoral as already mentioned: For a given sector, there are technological spillovers from one region to the other. Thanks to these spillovers, returns to scale at the sectoral level are constant. This is the simplest way to model spillovers. It is based on the Arrowian learning-by-investing mechanism, resuscitated by Romer (1986). Introducing R&D expenditures would have complicated unnecessarily the model given our main objectives. Rather we consider the shortcut of learning-by-investing to get a tractable yet far from trivial inter-sectoral inter-regional growth model. Boucekkine, del Rio and Licandro (2003) have already studied two-sector models with learning-by-investing in each sector. However, they consider a non-regional closed-economy, which has its advantages and its disadvantages from the analytical point of view. Given the environmental motivation of the paper, it seems out of question that the open economy structure adopted here is more adapted.

Within our framework, the respective specializations of the two regions converge through time, but not necessarily in a monotonous way. Following Böhringer and Lange (2005), the permits endowment of a firm is a function of its past emissions, and according to the scenario, this feature is internalized by the firm or not. The impact of the environmental policy on the sectors with respect to the baseline (no policy) scenario is compared successively at the level of (i) the sectors' growth rate, (ii) the sectors' capital stock, (iii) the sectors' added value, (iv) the sectors' revenue after transfers (i.e. taking account of the permits endowments), (v) the regions' added value, and (vi) the regions' revenue after transfers.

We now summarize the principal results. At the level of the growth rate and of the capital stock,

- (i) a first result is that a given sector is identically affected in both regions. This follows from the fact that regions face the same exogenous prices and share the same technologies.
- (ii) Given an "emission-based grand-fathering" sharing rule, a second result is that the energy intensive sector is more (negatively) affected by the environmental policy than the other sector.

To evaluate how results depends on the chosen endowment rule, we consider a dynamic *production*-based grandfathering rule, where a firm's current permits endowments depends on its previous period production. And we indeed show that result (ii) above does not necessarily hold with such a "production-based grand-fathering" sharing rule.

The impact of the environmental policy at the sectoral added value level results from (i) its impact at the capital level identified in the previous paragraph and (ii) a "price

effect” linked to the increase of the total cost of energy. This second effect is favorable to the energy intensive sector because the elasticity of its energy consumption to the total cost of energy is higher. Thus the comparison of the impacts on the sectoral added value is not trivial. Nevertheless, in the case of an ”emission-based grand-fathering” sharing rule, one verifies that the impact at the level of capital is likely to prevail after some time. This will in particular happen in the case of an economy submitted to a more and more constraining environmental policy driving up the ratio price of permits/price of energy.

The total sectoral revenue is defined as the sectoral added value plus the net transfer received by the sector. This net transfer is equal to the endowment of permits received less the permits used, multiplied by the price of permits. Because the baseline is characterized by sectoral growth rates that converge to the same limit, and under the assumption that no sector is advantaged w.r.t. the other (i.e. the relation between permits endowment and energy consumption is the same for the two sectors), then the energy intensive sector is at least after some time more affected by the environmental policy than the other sector. It also appears that the fact that a firm internalize or not the relation between its permits endowment and its past emissions does not affect this last result.

The last stage of the analysis is to evaluate the impact of the environmental policy at the regional level. This is done both at the level of the regional added value and at the level of the total regional revenue (i.e. taking account of the regional endowments of permits). The impact depends crucially on the specialization of the region along the baseline. The two regions converge to the same specialization (measured by the ratio of the capital stocks of their respective sectors), but not necessarily in a monotonous way. Conditions are established under which the spread of specialization, that depends on the initial capital endowments and on the technological spillovers, might be reversed. If the energy intensive sector is more affected than the other sector, then the region specialized in the energy intensive sector suffers more from the environmental policy. One also observes that translated in the framework of the Belgian debate, our result suggests that there will be no inversion of specialization (at least after some time), so that Wallonia is likely to be more affected than Flanders by environmental policies as modeled in this paper, not only in the short run (as in Germain et al., 2006) but also in the long run.

The structure of the paper is the following. Section 2 characterizes (i) the dynamic 2 sectors - 2 regions - 2 inputs (capital and energy) Heckscher-Ohlin model of a small open economy, (ii) the way interregional technological spillovers at the sector level are modeled and (iii) the specialization of the regions. Section 3 is devoted to the impacts of an environmental policy in the framework of an international permits market. One starts by characterizing the main assumptions underlying the baseline (no policy) scenario. Then the impacts of the environmental policy are evaluated as stated above, starting at the level of the sectors’ growth rate and finishing at the level of the regions’ total revenue.

## 2 The model

We model a 2 regions - 2 sectors - 2 inputs small open multiregional economy where :

- the 2 regions are indexed by  $i$  ( $i = v, w$ ),
- the 2 sectors (or 2 goods) are indexed by  $j$  ( $j = a, b$ ),
- the 2 inputs are capital ( $k$ ) and fossil energy ( $e$ ).

Capital is understood in a broad sense, i.e. as a bundle of inputs like physical and human capital, infrastructures, non-fossil energy,... Given that we consider a small open economy, agents are price-takers and prices are determined by the Rest of the World and thus exogenous. Sector  $a$  produces capital goods, sector  $b$  produces consumption goods. Energy is imported. Technology depends only on the sector and is the same for the two regions. National and foreign products of a certain type are supposed to be perfect substitutes. Emissions are linked to energy consumption, and for the sake of simplicity,  $e$  denotes simultaneously energy and emissions. We assume that there exists an international tradable permits market where polluting firms can buy or sell permits at a given exogenous price.

### 2.1 Behavior of the firms

The technology of sector  $j$  ( $j = a, b$ ) of region  $i$  ( $i = v, w$ ) at date  $t$  ( $t \geq 1$ ) is described by the following Cobb-Douglas production function with decreasing returns to scale

$$y_{ijt} = A_{ijt} k_{ijt}^{\alpha_j} e_{ijt}^{\beta_j} \quad (1)$$

where  $y$ ,  $k$  and  $e$  are production, capital and energy respectively and  $A$  is a coefficient that measures technological progress. The technological parameters  $\alpha_j$  and  $\beta_j$  verify  $0 < \alpha_j, \beta_j < \alpha_j + \beta_j < 1$ .

The representative firm of sector  $j$  of region  $i$  is assumed to choose the flow of its energy consumption and investment in order to maximise the sum of its discounted profits :

$$\Pi_{ij} = \max_{\{e_{ijt}, i_{ijt}\}_{t \geq 1}} \sum_{t \geq 1} \frac{\pi_{ijt}}{[1+r]^t} \quad (2)$$

under the constraints that

$$k_{ijt} = k_{ij,t-1}[1 - \delta_j] + i_{ijt}, \quad t \geq 1 \quad (3)$$

$$\bar{e}_{ijt} = \tilde{e}_{ijt} + \lambda_{jt} e_{ij,t-1} + \mu_{jt} y_{ij,t-1}, \quad t \geq 1 \quad (4)$$

where by definition

$$\pi_{ijt} = p_{jt} y_{ijt} - q_t e_{ijt} - p_{at} i_{ijt} + \tau_t [\bar{e}_{ijt} - e_{ijt}] \quad (5)$$

and where  $e_{ij0}$ ,  $k_{ij0}$  and  $y_{ij0}$  are given.  $r$  is the (exogenous) positive discount rate. (3) is the familiar capital accumulation equation, where  $\delta$  measures the depreciation rate of capital and (5) defines  $\pi_{ijt}$  as the current profit of sector  $j$  of region  $i$  at date  $t$ , where  $p_j, q$  and  $\tau$  are the prices of good  $j$  ( $j = a, b$ ), energy and tradable permits respectively. Following Böringer and Lange (2005), (4) defines the permits endowment received by sector  $j$  of region  $i$  at date  $t$  ( $\bar{e}_{ijt}$ ) as a linear function of its production and emissions of the previous period, where  $\tilde{e}_{ijt}$ ,  $\lambda_{jt}$  and  $\mu_{jt}$  are given exogenous (eventually variable) positive parameters.

After substitution of (5), (3) and (4) in (2), the problem rewrites

$$\begin{aligned}
& \max_{\{e_{ijt}, i_{ijt}\}_{t \geq 1}} \sum_{t \geq 1} \frac{1}{[1+r]^t} [p_{jt} y_{ijt} - [q_t + \tau_t] e_{ijt} \\
& - p_{at} [k_{ijt} - k_{ij,t-1} [1 - \delta_j]] + \tau_t [\tilde{e}_{ijt} + \lambda_{jt} e_{ij,t-1} + \mu_{jt} y_{ij,t-1}]] \\
= & \frac{1}{[1+r]} [p_{a1} k_{ij0} [1 - \delta_j] + \tau_1 [\lambda_{j1} e_{ij0} + \mu_{j1} y_{ij0}]] \\
& + \max_{\{e_{ijt}, i_{ijt}\}_{t \geq 1}} \sum_{t \geq 1} \frac{1}{[1+r]^t} \left[ \left[ p_{jt} + \tau_{t+1} \frac{\mu_{j,t+1}}{1+r} \right] y_{ijt} - \left[ q_t + \tau_t - \tau_{t+1} \frac{\lambda_{j,t+1}}{1+r} \right] e_{ijt} \right. \\
& \left. - \left[ p_{at} - p_{a,t+1} \frac{1 - \delta_j}{1+r} \right] k_{ijt} + \tau_t \tilde{e}_{ijt} \right] \\
= & C^{te} + \max_{\{e_{ijt}, i_{ijt}\}_{t \geq 1}} \sum_{t \geq 1} \frac{1}{[1+r]^t} [\hat{p}_{jt} y_{ijt} - \tilde{q}_t e_{ijt} - \tilde{p}_{jt} k_{ijt} + \tau_t \tilde{e}_{ijt}] \tag{6}
\end{aligned}$$

where we define

$$\hat{p}_{jt} = p_{jt} + \tau_{t+1} \frac{\mu_{j,t+1}}{1+r} \tag{7}$$

as the *adjusted price* of sector  $j$  at date  $t$ ;

$$\tilde{p}_{jt} = \left[ p_{at} - p_{a,t+1} \frac{1 - \delta_j}{1+r} \right] \tag{8}$$

as the *user cost of capital* of sector  $j$  at date  $t$  and

$$\tilde{q}_{jt} = q_t + \tau_t - \tau_{t+1} \frac{\lambda_{j,t+1}}{1+r} \tag{9}$$

as the *user cost of energy* of sector  $j$  at date  $t$ .

First order conditions lead to

$$\begin{aligned}
\frac{\partial \Pi_{ij}}{\partial k_{ijt}} &= \alpha_j \hat{p}_{jt} A_{ijt} k_{ijt}^{\alpha_j - 1} e_{ijt}^{\beta_j} - \tilde{p}_{at} = 0 \\
\frac{\partial \Pi_{ij}}{\partial e_{ijt}} &= \beta_j \hat{p}_{jt} A_{ijt} k_{ijt}^{\alpha_j} e_{ijt}^{\beta_j - 1} - \tilde{q}_{jt} = 0
\end{aligned}$$

which leads to the following solutions

$$k_{ijt} = \left[ p_{jt} A_{ijt} \left[ \frac{\alpha_j}{\tilde{p}_{jt}} \right]^{1-\beta_j} \left[ \frac{\beta_j}{\tilde{q}_{jt}} \right]^{\beta_j} \right]^{\frac{1}{1-\alpha_j-\beta_j}} \tag{10}$$

$$e_{ijt} = \left[ p_{jt} A_{ijt} \left[ \frac{\alpha_j}{\tilde{p}_{jt}} \right]^{\alpha_j} \left[ \frac{\beta_j}{\tilde{q}_{jt}} \right]^{1-\alpha_j} \right]^{\frac{1}{1-\alpha_j-\beta_j}} \tag{11}$$

$$y_{ijt} = \left[ p_{jt}^{\alpha_j + \beta_j} A_{ijt} \left[ \frac{\alpha_j}{\tilde{p}_{jt}} \right]^{\alpha_j} \left[ \frac{\beta_j}{\tilde{q}_{jt}} \right]^{\beta_j} \right]^{\frac{1}{1-\alpha_j-\beta_j}} \tag{12}$$

The previous formulas lead to



$$e_{ijt} = \frac{\beta_j \widehat{p}_{jt} y_{ijt}}{\widetilde{q}_{jt}} \quad (13)$$

$$y_{ijt} = \frac{\widetilde{p}_j k_{ijt}}{\alpha_j \widehat{p}_{jt}} \quad (14)$$

## 2.2 Endogenous growth and technical spillovers

One assumes that

$$A_{ijt} = \rho_t [\theta_{jt} k_{ij,t-1} + [1 - \theta_{jt}] K_{j,t-1}]^{\gamma_j} \quad (15)$$

where by definition,  $K_{jt} = k_{wjt} + k_{vjt}$  is the capital of sector  $j$  at the country level.  $\gamma_j$  is such that  $\alpha_j + \beta_j + \gamma_j = 1$ , i.e. global returns to scale are constant.  $\rho_t$  and  $\theta_{jt}$  ( $t \geq 1$ ) are exogenous positive parameters. With respect to the spillover parameters  $\theta_{jt}$ , we assume that  $0 \leq \theta_{jt} < 1$  ( $\forall t \geq 1$ ) and  $\lim_{t \rightarrow +\infty} \theta_{jt} = \overline{\theta}_j < 1$ . (15) shows that the productivity factor of sector  $j$  of region  $i$  at time  $t$  depends not only of the capital of this sector inherited from the previous period, but also of the capital of the same sector of the other region. There is thus a interregional technological spillover at the sector level. The spillover is intra-sectoral as mentioned repeatedly below. Notice that the larger  $\theta_{jt}$ , the lower the impact of the learning-by-investing accumulated in region  $w$  on technological progress in region  $v$  for sector  $j$ . We shall use this observation in some interpretations later on.

Finally, we assume that the price of sector  $a$  is the numeraire, i.e.  $p_{at} = 1$  ( $\forall t \geq 1$ ), which implies that the cost of capital is constant for both sectors ( $\widetilde{p}_{jt} = \widetilde{p}_j = \frac{\delta_j + r}{1+r}$ ,  $j = a, b$ ).

Given the previous assumptions, and given (10), one obtains

$$\begin{aligned} k_{ijt} &= \left[ \widehat{p}_{jt} \left[ \frac{\alpha_j}{\widetilde{p}_j} \right]^{1-\beta_j} \left[ \frac{\beta_j}{\widetilde{q}_{jt}} \right]^{\beta_j} \right]^{\frac{1}{1-\alpha_j-\beta_j}} A_{ijt}^{\frac{1}{1-\alpha_j-\beta_j}} \\ &= \zeta_{jt} \rho_t [\theta_{jt} k_{ij,t-1} + [1 - \theta_{jt}] K_{j,t-1}] \\ &= \zeta_{jt} \rho_t [k_{ij,t-1} + [1 - \theta_{jt}] k_{\widetilde{i}j,t-1}] \end{aligned} \quad (16)$$

where  $\widetilde{i}$  designates the other region and where

$$\zeta_{jt} =_{def} \left[ \widehat{p}_{jt} \left[ \frac{\alpha_j}{\widetilde{p}_j} \right]^{1-\beta_j} \left[ \frac{\beta_j}{\widetilde{q}_{jt}} \right]^{\beta_j} \right]^{\frac{1}{1-\alpha_j-\beta_j}} \quad (17)$$

For each sector  $j$ , we consider the system:

$$k_{wjt} = \zeta_{jt} \rho [k_{wj,t-1} + [1 - \theta_{jt}] k_{vj,t-1}] \quad (18)$$

$$k_{vjt} = \zeta_{jt} \rho_t [k_{vj,t-1} + [1 - \theta_{jt}] k_{wj,t-1}] \quad (19)$$

$t \in \mathcal{T}$ ,  $k_{wj0}$  and  $k_{vj0}$  given.

**Proposition 1** *The solutions to the system (18-19) can be expressed as:*

$$k_{wjt} = \frac{1}{2} \prod_{m=1}^t \zeta_{jm} \rho_{jm} \left\{ [k_{wj0} + k_{vj0}] \prod_{m=1}^t [2 - \theta_{jm}] + [k_{wj0} - k_{vj0}] \prod_{m=1}^t \theta_{jm} \right\} \quad (20)$$

$$k_{vjt} = \frac{1}{2} \prod_{m=1}^t \zeta_{jm} \rho_{jm} \left\{ [k_{wj0} + k_{vj0}] \prod_{m=1}^t [2 - \theta_{jm}] + [k_{vj0} - k_{wj0}] \prod_{m=1}^t \theta_{jm} \right\} \quad (21)$$

**Proof.** See appendix.  $\square$

It follows immediately that for a given sector  $j$ , if the two regions have the same initial endowment of capital in that sector (i.e.  $k_{wj0} = k_{vj0}$ ), then their respective endowment of capital remain equal in all periods ( $k_{vjt} = k_{wjt} = \frac{1}{2} [k_{wj0} + k_{vj0}] \prod_{m=1}^t \zeta_{jm} \rho_{jm} [2 - \theta_{jm}]$ ).

Furthermore :

**Corollary 1** *Whatever the initial conditions and the exogenous patterns, the regional capital stocks of the same sector converge to the same pattern asymptotically :*

$$\lim_{t \rightarrow +\infty} k_{wjt} = \lim_{t \rightarrow +\infty} k_{vjt} = \frac{1}{2} [k_{wj0} + k_{vj0}] \prod_{m=1}^{+\infty} \zeta_{jm} \rho_{jm} [2 - \theta_{jm}], \quad j = a, b$$

**Proof.** By Proposition 1, one can write for Region  $i$  :

$$k_{ijt} = \frac{1}{2} \left\{ \prod_{m=1}^t \zeta_{jm} \rho_{jm} [2 - \theta_{jm}] \right\} \left\{ k_{ij0} + \tilde{k}_{ij0} + [k_{ij0} - \tilde{k}_{ij0}] \prod_{m=1}^t \frac{\theta_{jm}}{2 - \theta_{jm}} \right\} \quad (22)$$

where  $\tilde{i}$  is the other region. Given that by assumption, the sequence  $\theta_{jt}$  is such that  $0 \leq \theta_{jt} < 1$  and  $\lim_{t \rightarrow +\infty} \theta_{jt} = \bar{\theta}_j < 1$ , we have :  $\frac{\theta_{jm}}{2 - \theta_{jm}} < 1, \forall m \geq 1$ , so that the sequence  $\omega_{jt} = \prod_{m=1}^t \frac{\theta_{jm}}{2 - \theta_{jm}}$  is strictly decreasing, and since it is bounded, it is converging. Notice that  $\omega_{jt}$  is asymptotically geometric with coefficient  $\frac{\bar{\theta}_j}{2 - \bar{\theta}_j} < 1$ , which implies that  $\lim_{t \rightarrow +\infty} \omega_{jt} = 0$ .  $\square$

This strong result follows from the fact that besides different initial sectoral dotation of capital, the two regions are identical : they face the same international prices and share the same technologies. In the presence of intra-sectoral spillovers, the divergence force coming from endogenous growth (namely, constant returns) is neutralized. Indeed, notice that if the parameters  $\theta_{jt} = 1$  for every  $t$ , then equations (18) and (19) would imply that the capital stocks of the two regions will diverge over time if the initial stocks are different. If this sequence of parameters is permanently strictly below 1, then such a divergence vanishes because intra-sectoral spillovers will neutralize the divergence force arising from the underlying AK structure.

Given (22), one can express the growth rate of sector  $j$  of region  $i$  at time  $t$  as :

$$g_{ijt} = \frac{k_{ijt}}{k_{ij,t-1}} = \zeta_{jt} \rho_{jt} [2 - \theta_{jt}] X_{ijt}, \quad i = v, w, \quad j = a, b, \quad t \geq 1 \quad (23)$$

where by definition :

$$X_{ijt} = \frac{k_{ij0} + \tilde{k}_{ij0} + [k_{ij0} - \tilde{k}_{ij0}] \prod_{m=1}^t \frac{\theta_{jm}}{2 - \theta_{jm}}}{k_{ij0} + \tilde{k}_{ij0} + [k_{ij0} - \tilde{k}_{ij0}] \prod_{m=1}^{t-1} \frac{\theta_{jm}}{2 - \theta_{jm}}} \quad (24)$$

In the long term,  $g_{ijt}$  tends to  $\zeta_{jt} \rho_{jt} [2 - \bar{\theta}_j]$ , which as expected does not depend on  $i$ .

## 2.3 Specialization of the regions

Let us define the *specialization index* of Region  $i$  ( $i = v, w$ ) at time  $t$  by the ratio :

$$\chi_{it} = \frac{k_{iat}}{k_{ibt}} \quad (25)$$

We also define the *spread of specialization index* at time  $t$  by the ratio of the specialization indexes of the two regions :

$$\sigma_t = \frac{\chi_{wt}}{\chi_{vt}} = \frac{k_{wat} k_{vbt}}{k_{wbt} k_{vat}}$$

The purpose of the present subsection is to study what determines the evolution of the regions' specialization through time. Using (22), we have :

$$\chi_{it} = \frac{k_{iat}}{k_{ibt}} = \frac{\left\{ \prod_{m=1}^t \zeta_{am} \rho_m [2 - \theta_{jm}] \right\} \left\{ k_{ia0} + \tilde{k}_{ia0} + [k_{ia0} - \tilde{k}_{ia0}] \prod_{m=1}^t \frac{\theta_{am}}{2 - \theta_{am}} \right\}}{\left\{ \prod_{m=1}^t \zeta_{bm} \rho_m [2 - \theta_{jm}] \right\} \left\{ k_{ib0} + \tilde{k}_{ib0} + [k_{ib0} - \tilde{k}_{ib0}] \prod_{m=1}^t \frac{\theta_{bm}}{2 - \theta_{bm}} \right\}}, \quad i = v, w$$

and

$$\sigma_t = \frac{\left\{ k_{wa0} + k_{va0} + [k_{wa0} - k_{va0}] \prod_{m=1}^t \frac{\theta_{am}}{2 - \theta_{am}} \right\} \left\{ k_{wb0} + k_{vb0} + [k_{wb0} - k_{vb0}] \prod_{m=1}^t \frac{\theta_{bm}}{2 - \theta_{bm}} \right\}}{\left\{ k_{wb0} + k_{vb0} + [k_{wb0} - k_{vb0}] \prod_{m=1}^t \frac{\theta_{bm}}{2 - \theta_{bm}} \right\} \left\{ k_{wa0} + k_{va0} + [k_{wa0} - k_{va0}] \prod_{m=1}^t \frac{\theta_{am}}{2 - \theta_{am}} \right\}} \quad (26)$$

One observes the remarkable fact that the spread of specialization index depends only on the initial stocks of capital and on the spillovers parameters that are exogenous. As a consequence this index will not be affected by the environmental policies that are considered below. This does not mean that the specialization index of a region (defined by (25)) does not change under an environmental policy w.r.t. the baseline. It means that the specialization indexes of the two regions are affected identically such that their spread remains unchanged.

In the *long run*, these formulas imply the following properties :

$$\lim_{t \rightarrow +\infty} \chi_{it} = \frac{K_{a0}}{K_{b0}} \left[ \prod_{m=1}^{+\infty} \frac{\zeta_{am}}{\zeta_{bm}} \right], \quad i = v, w \quad (27)$$

and

$$\lim_{t \rightarrow +\infty} \sigma_t = 1 \quad (28)$$

This result follows immediately from Corollary 1. (27) shows that in the long run the regional specialization index reflects the initial specialization index at the *national* level and the exogenous patterns of prices (through the parameters  $\zeta_{jm}$  (see (17)), which interact multiplicatively in our model.

Assume that  $\sigma_0 > 1$  (i.e. Region  $w$  is more specialized in the production of the capital good). The preceding result does not imply that  $\sigma_t$  decreases monotonously from  $\sigma_0 > 1$  to  $\sigma_\infty = 1$ . We have indeed the following proposition :

**Proposition 2** *The spread of specialization index might be reversed in period  $t$  w.r.t. period 0 (i.e.  $\sigma_0 > 1$  and  $\sigma_t < 1$ ), depending on the ratio of the spillover parameters  $\theta_{bt}/\theta_{at}$  ( $t \geq 1$ ) and on the initial capital endowments.*

**Proof.** Given (26), it is easy to verify that  $\sigma_t < 1$  implies that :

$$[k_{wb0} + k_{vb0}] [k_{wa0} - k_{va0}] \prod_{m=1}^t \frac{\theta_{am}}{2 - \theta_{am}} < [k_{wa0} + k_{va0}] [k_{wb0} - k_{vb0}] \prod_{m=1}^t \frac{\theta_{bm}}{2 - \theta_{bm}}$$

Two cases emerge :

(a)  $k_{wb0} - k_{vb0} > 0$  : then it follows that  $k_{wa0} - k_{va0} > 0$  because  $\sigma_0 > 1$  by assumption. Then  $\sigma_t < 1$  iff

$$\frac{[k_{wb0} + k_{vb0}] [k_{wa0} - k_{va0}]}{[k_{wa0} + k_{va0}] [k_{wb0} - k_{vb0}]} < \prod_{m=1}^t \frac{\frac{\theta_{bm}}{2 - \theta_{bm}}}{\frac{\theta_{am}}{2 - \theta_{am}}} \quad (29)$$

which holds if the sequence  $\{\theta_{bt}, t \geq 1\}$  is in "average" high enough w.r.t. the sequence  $\{\theta_{at}, t \geq 1\}$ <sup>2</sup>;

(b)  $k_{wb0} - k_{vb0} < 0$  : it follows that  $\sigma_t < 1$  iff

$$\frac{[k_{wb0} + k_{vb0}] [k_{wa0} - k_{va0}]}{[k_{wa0} + k_{va0}] [k_{wb0} - k_{vb0}]} > \prod_{m=1}^t \frac{\frac{\theta_{bm}}{2 - \theta_{bm}}}{\frac{\theta_{am}}{2 - \theta_{am}}} \quad (30)$$

which holds if  $k_{wa0} - k_{va0} < 0$  and if the sequence  $\{\theta_{bt}, t \geq 1\}$  is in "average" low enough w.r.t. the sequence  $\{\theta_{at}, t \geq 1\}$ <sup>3</sup>.  $\square$

### 3 Impact of an environmental policy

#### 3.1 Preliminaries

We assume that the baseline tends in the long run to a balanced growth path characterized by the asymptotic constant growth rate  $g$  that applies to both sectors. Formally, this means that

$$g_{ijt}^B = \frac{k_{ijt}^B}{k_{ij,t-1}^B} = \zeta_{jt}^B \rho_t [2 - \theta_{jt}] X_{ijt} = g X_{ijt} \quad (31)$$

where given (17),

$$\zeta_{jt}^B = \left[ p_{jt} \left[ \frac{\alpha_j}{\tilde{p}_j} \right]^{1-\beta_j} \left[ \frac{\beta_j}{q_t} \right]^{\beta_j} \right]^{\frac{1}{1-\varphi}} \quad (32)$$

and where  $X_{ijt}$  is defined by (24). This is equivalent to assume that at the world level, the evolution of prices of goods reflects sectoral technical progress, the costs of inputs and technological spillovers. Because the price of sector  $a$  is the numéraire, this implies that

$$q_t = \left[ C_a \left[ \frac{\rho_t [2 - \theta_{at}]}{g} \right]^{1-\varphi} \right]^{1/\beta_a} \text{ and } p_{bt} = \frac{q_t^{\beta_b}}{C_b} \left[ \frac{g}{\rho_t [2 - \theta_{bt}]} \right]^{1-\varphi}, \text{ where } C_j = \left[ \frac{\alpha_j}{\tilde{p}_j} \right]^{1-\beta_j} \beta_j^{\beta_j} \ (j = a, b)$$

are constants.

As we aim to compare the impacts of an environmental policy (EP) between sectors and between regions, we state the two following important assumptions :

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<sup>2</sup>This implies that in the long term (i.e. when the sequences are close to their limit values  $\bar{\theta}_j$  ( $j = a, b$ )) the ratio  $\bar{\theta}_b/\bar{\theta}_a$  is high enough.

<sup>3</sup>In case (b), if  $k_{wa0} - k_{va0} > 0$ , then the regions have opposite specializations (i.e.  $k_{wa0} > k_{va0}$  and  $k_{wb0} < k_{vb0}$ ). Then a remarkable fact is that the previous inequality cannot be satisfied whatever the spillover parameters (inversion of specialization is impossible).

- Sector  $a$  is more energy intensive than sector  $b$  :

$$\beta_a > \beta_b \quad (33)$$

- Region  $w$  est initially more specialized in the production of good  $a$  than Region  $v$  :

$$\frac{k_{wa0}}{k_{wb0}} > \frac{k_{va0}}{k_{vb0}} \quad (34)$$

In this respect, assumption (34) means that Region  $w$  is initially more specialized than Region  $v$  in the production of good  $a$  ( $\chi_{w0} > \chi_{v0} \Leftrightarrow \sigma_0 > 1$ ).

We look at the impact of an environmental policy (EP) characterized by a sequence of permits prices  $\{\tau_t, t \geq 1\}$  and by permits endowments defined by (4). Now, as will soon become clear below, it is impossible to obtain clear-cut results when comparing the impacts of the EP between sectors or between regions if this policy is characterized by a general burden sharing rule like (4). In order to derive precise results, we will mainly focus on *dynamic emission-based grand-fathering* rules, where a firm's current permits endowment depends on its previous period emissions<sup>4</sup>. We qualify this dependance as the "endowment" effect and consider two cases whether this effect is internalized or not by firms when they maximise their profits :

- **No internalisation**<sup>5</sup> : one applies (4) with  $\lambda_{jt} = \mu_{jt} = 0$  ( $j = a, b, \forall t \geq 1$ ) and :

$$\bar{e}_{ijt} = \tilde{e}_{ijt} = \eta e_{ij,t-1},$$

where  $0 < \eta < 1$  and  $e_{ij,t-1}$  are the emissions characterising the EP until  $t - 1$ .  $\tilde{e}_{ijt}$  is considered as exogenous by the firms when they solve problem (2), then given (7) and (9),  $\hat{p}_{jt} = p_{jt}$  and  $\tilde{q}_{jt} = q_t + \tau_t$ .

- **Internalisation** : one applies (4) with  $\tilde{e}_{ijt} = \mu_j = 0$  and :

$$\bar{e}_{ijt} = \lambda_j e_{ij,t-1}$$

where  $0 < \lambda_j < 1$  and  $e_{ij,t-1}$  are the emissions characterising the EP until  $t - 1$ .  $\bar{e}_{ijt}$  is considered as endogenous by the firms when they solve problem (2), then given (7) and (9),  $\hat{p}_{jt} = p_{jt}$  and  $\tilde{q}_{jt} = q_t + \tau_t - \tau_{t+1} \frac{\lambda_j}{1+r}$ .

We will also assume that  $\lambda_j$  is sufficiently small so that the effect of today's energy consumption on tomorrow's permits endowment is never sufficient to counteract the direct increase of the total cost of energy through the permits price (i.e.  $\tau_t - \tau_{t+1} \frac{\lambda_j}{1+r} > 0$ ,  $j = a, b, \forall t \geq 1$ ). The baseline is characterized by a sequence of permits prices equal to 0.

In a next subsection, we will also briefly consider a "production-based grand-fathering" sharing rule in order to highlight how results can depend crucially on the chosen endowment rule.

### 3.2 Impact on the sectoral growth rates

Given (23), (31), (17) and (32), it follows that :

$$\frac{g_{ijt}}{g_{ijt}^B} = \frac{\zeta_{jt}}{\zeta_{jt}^B} = \left[ \frac{\tilde{p}_{jt}}{p_{jt}} \left[ \frac{q_t}{\tilde{q}_{jt}} \right]^{\beta_j} \right]^{\frac{1}{1-\varphi}} \quad (35)$$

<sup>4</sup>The dependance could go further in the past (see Böhringer and Lange (2005)).

<sup>5</sup>This could be justified if the rule applies at an aggregate level and if the firm is small and receives a fixed share of the total permits endowment.

A first observation is that the impact of the EP on the sectoral growth rates is the same in both regions, i.e. :

$$\frac{g_{wjt}}{g_{wjt}^B} = \frac{g_{vjt}}{g_{vjt}^B}, \quad j = a, b \quad (36)$$

This is because the sectors  $j$  of both regions face the same exogenous prices and share the same technologies.

Given assumptions (33) and (34) and the assumptions about the EP in the previous subsection, (35) can be rewritten :

$$\frac{g_{ijt}}{g_{ijt}^B} = \left[ \frac{1}{1 + \frac{\tau_t - \tau_{t+1} \frac{\lambda_j}{1+r}}{q_t}} \right]^{\frac{\beta_j}{1-\varphi}} \quad (37)$$

A second observation is that the EP affects negatively the sectoral growth rates, i.e.  $g_{ijt} < g_{ijt}^B$ , because the user cost of energy increases (given that we have assumed that  $\tau_t - \tau_{t+1} \frac{\lambda_j}{1+r} > 0, \forall j = a, b; \forall t \geq 1$ ).

We now compare the impacts of the EP on the capital accumulation of the two sectors of a given region w.r.t. the baseline. More precisely, we look in what extent the following inequality is verified :

$$\frac{g_{iat}}{g_{iat}^B} < \frac{g_{ibt}}{g_{ibt}^B}, \quad i = v, w \quad (38)$$

By assumption (33) ( $\beta_a > \beta_b$ ), and thanks to the fact that it is assumed that the user cost of energy increases with the EP, it follows from (37) that a sufficient condition for the inequality (38) to be verified is that  $\lambda_a \leq \lambda_b$ . The growth rate of the energy intensive capital sector  $a$  is more affected by the EP than the accumulation capital of the other sector if the endowment effect relative to sector  $a$  is not higher than the one relative to sector  $b$ .

### 3.3 Impact on the sectoral capital stock

We compare the impacts of the EP on the capital accumulation of the two sectors of a given region w.r.t. the baseline. More precisely, we look in what extent the following inequality is verified :

$$\frac{k_{iat}}{k_{iat}^B} < \frac{k_{ibt}}{k_{ibt}^B}, \quad i = v, w \quad (39)$$

Given (23) and (31), one has :

$$k_{ijt} = k_{ij0} \prod_{m=1}^t g_{ijm} \quad \text{and} \quad k_{ijt}^B = k_{ij0} \prod_{m=1}^t g_{ijm}^B$$

so that :

$$\frac{k_{ijt}}{k_{ijt}^B} = \prod_{m=1}^t \frac{g_{ijm}}{g_{ijm}^B} = \prod_{m=1}^t \left[ \frac{1}{1 + \frac{\tau_m - \tau_{m+1} \frac{\lambda_j}{1+r}}{q_m}} \right]^{\frac{\beta_j}{1-\varphi}} \quad (40)$$

where the second equality follows from (37). By the previous subsection, we know that a sufficient condition for the inequality  $g_{ijm} < g_{ijm}^B$  ( $\forall j = a, b; \forall t \geq 1$ ) to be verified is that  $\lambda_a \leq \lambda_b$ . Then it follows immediately that the accumulation of capital of the energy

intensive capital sector  $a$  is more affected by the EP than the accumulation capital of the other sector if the endowment effect relative to sector  $a$  is not higher than the one relative to sector  $b$ .

Furthermore, given (36), it follows that the accumulation of capital of a given sector is affected in the same way in both regions, i.e. :

$$\frac{k_{wjt}}{k_{wjt}^B} = \frac{k_{vjt}}{k_{vjt}^B}, \quad j = a, b \quad (41)$$

### 3.4 The dynamic production-based grand-fathering rule

To highlight how results depends crucially on the chosen endowment rule, let us consider briefly the *dynamic production-based grand-fathering* rule, where a firm's current permits endowment depends on its previous period production<sup>6</sup>. We assume that this dependance is identical across sectors and regions and that it is internalized by firms when they maximise their profits. Formally, one applies (4) with  $\tilde{e}_{ijt} = \lambda_{jt} = 0$  and :

$$\bar{e}_{ijt} = \mu y_{ij,t-1}$$

where  $0 < \mu < 1$  and  $y_{ij,t-1}$  is the production characterising the EP until  $t - 1$ .  $\bar{e}_{ijt}$  is considered as endogenous by the firms when they solve problem (2). Then given (7) and (9),  $\hat{p}_{jt} = p_{jt} + \tau_{t+1} \frac{\mu}{1+r}$  and  $\tilde{q}_{jt} = q_t + \tau_t$  and (40) becomes

$$\frac{k_{ijt}}{k_{ijt}^B} = \prod_{m=1}^t \left[ \left[ 1 + \frac{\mu}{1+r} \frac{\tau_{m+1}}{p_{jm}} \right] \left[ \frac{1}{1 + \frac{\tau_m}{q_m}} \right]^{\beta_j} \right]^{\frac{1}{1-\varphi}}$$

Recall that given the assumptions about the baseline (cfr. subsection 3.1),

$q_t = \left[ C_a \left[ \frac{\rho_t [2-\theta_{at}]}{g} \right]^{1-\varphi} \right]^{1/\beta_a}$ ,  $p_{at} = 1$  and  $p_{bt} = \frac{q_t^{\beta_b}}{C_b} \left[ \frac{g}{\rho_t [2-\theta_{bt}]} \right]^{1-\varphi}$  (with  $C_j = \left[ \frac{\alpha_j}{\tilde{p}_j} \right]^{1-\beta_j} \beta_j^{\beta_j}$  ( $j = a, b$ )). Because of assumption (33), the second term between brackets of the RHS is lower for sector  $a$ . But if  $p_{bt}$  is increasing (as is observed empirically), then the first term between brackets of the RHS is lower for sector  $b$ , and the overall effect is undecidable without more information on the parameters. Configurations of parameters are possible where one obtains the opposite effect as the one obtained with the emission-based grand-fathering rules analyzed above, i.e. with equality (39) is reversed (for example if the consumption good price increases sufficiently quickly because of a low exogenous technical progress  $\rho_t$ ).

### 3.5 Impact on the sectoral added value

In the remaining of the paper we return exclusively to the emission-based grand-fathering rules because they lead to more clear-cut conclusions. To simplify the analysis, we make the additional assumption that the endowment effect is identical across sectors and regions (i.e.  $\lambda_j = \lambda$  ( $j = a, b$ )). The EP is thus characterized by  $\hat{p}_{jt} = p_{jt}$  and  $\tilde{q}_{jt} = \tilde{q}_t = q_m + \tau_m - \tau_{m+1} \frac{\lambda}{1+r}$ .

We now compare the impacts of the EP on the added value of the two sectors of a given region w.r.t. the baseline. In our setting, the intermediate consumption is limited

<sup>6</sup>The dependance could go further in the past (see Böhringer and Lange (2005)).

to the imported energy consumption. Thus :

$$va_{ijt} = p_{jt}y_{ijt} - q_t e_{ijt}$$

Given (13) and (14), the previous identity becomes :

$$va_{ijt} = \left[ \frac{p_{jt}}{\widehat{p}_{jt}} - \beta_j \frac{q_t}{\widetilde{q}_{jt}} \right] \frac{\widetilde{p}_j k_{ijt}}{\alpha_j} \quad (42)$$

Along the baseline,  $\widehat{p}_{jt} = p_{jt}$  and  $\widetilde{q}_{jt} = q_t$ , so that

$$va_{ijt}^B = [1 - \beta_j] \frac{\widetilde{p}_j k_{ijt}^B}{\alpha_j} \quad (43)$$

The impact of the EP on the added value of sector  $j$  of region  $i$  at time  $t$  is measured by the following ratio:

$$\frac{va_{ijt}}{va_{ijt}^B} = \frac{1 - \beta_j \frac{q_t}{\widetilde{q}_{jt}}}{1 - \beta_j} \frac{k_{ijt}}{k_{ijt}^B} \quad (44)$$

Because the ratio  $\frac{k_{ijt}}{k_{ijt}^B}$  does not depend on  $i$  (recall (41)), this is also true for  $\frac{va_{ijt}}{va_{ijt}^B}$ .

In a similar manner as what was done for the capital stock, we look in what extent the following inequality is verified:

$$\frac{va_{iat}}{va_{iat}^B} < \frac{va_{ibt}}{va_{ibt}^B}, \quad i = v, w \quad (45)$$

As shown by (44), the global impact on  $va_{ijt}$  is the product of two fractions. The second one measures the impact on the regional sectoral capital stock analyzed in the previous subsection. We know that this impact is stronger for sector  $a$ . The first fraction measures an effect due to the increase of the total cost of energy. Unfortunately, this energy cost effect affects the added value the other way round ( $1 - \beta_j \frac{q_t}{\widetilde{q}_{jt}} > 1 - \beta_j$ ). Indeed, because of the increase of total cost of energy, its share arises in the value of production. And this effect is stronger the higher  $\beta_j$ . Thus the inequality (45) does not follow trivially.

The substitution of (40) and (44) in (45) leads to:

$$\frac{1 - \beta_a \frac{1}{1 + \frac{\tau_t - \tau_{t+1} \frac{\lambda}{1+r}}{q_t}}}{1 - \beta_a} \prod_{m=1}^t \left[ \frac{1}{1 + \frac{\tau_m - \tau_{m+1} \frac{\lambda}{1+r}}{q_m}} \right]^{\frac{\beta_a}{1-\varphi}} < \frac{1 - \beta_b \frac{1}{1 + \frac{\tau_t - \tau_{t+1} \frac{\lambda}{1+r}}{q_t}}}{1 - \beta_b} \prod_{m=1}^t \left[ \frac{1}{1 + \frac{\tau_m - \tau_{m+1} \frac{\lambda}{1+r}}{q_m}} \right]^{\frac{\beta_b}{1-\varphi}} \quad (46)$$

One has the following proposition:

**Proposition 3** *Inequality (45) (or (46)) is likely to be satisfied if  $\prod_{m=1}^t \left[ \frac{1}{1 + \frac{\tau_m - \tau_{m+1} \frac{\lambda}{1+r}}{q_m}} \right]$  is small enough.*

**Proof.** See appendix.  $\square$

This should happen at least after some time because the sequence  $\left\{ \prod_{m=1}^t \left[ \frac{1}{1 + \frac{\tau_m - \tau_{m+1} \frac{\lambda}{1+r}}{q_m}} \right], t \geq 1 \right\}$  tends to 0. The above proposition is likely to be verified



in the case of a more and more constraining EP. Then the price of permits  $\tau$  should indeed increase. This would induce firms to reduce their demand of fossil energy, thereby pushing down the price of fossil energy  $q$ .

On the contrary, the above proposition is likely to be invalidated if the EP remains loose and if increasing scarcity pushes the price of energy up. Indeed the sequence

$\left\{ \prod_{m=1}^t \left[ \frac{1}{1 + \frac{\tau_m - \tau_{m+1} \frac{\lambda}{1+r}}{q_m}} \right], t \geq 1 \right\}$  diverges if  $\lim_{m \rightarrow +\infty} \frac{\tau_m - \tau_{m+1} \frac{\lambda}{1+r}}{q_m} = 0$ , which would only happen if the price of fossil energy increases at a higher rate than the price of permits.

### 3.6 Impact on the sectoral revenues with transfers

The sectoral revenue with transfers is defined as the sum of the added value of the sector and of the net endowment of permits:

$$r_{ijt} = va_{ijt} + \tau_t [\bar{e}_{ijt} - e_{ijt}], \quad i = v, w, \quad j = a, b, \quad t \geq 1 \quad (47)$$

where  $\bar{e}_{ijt}$  is defined by (4).

Let us first consider the non anticipated emission-based grand-fathering sharing rule. Given (47) and the fact that along the baseline,  $r_{ijt}^B = va_{ijt}^B$ , it follows that:

$$\frac{r_{ijt}}{r_{ijt}^B} = \frac{va_{ijt}}{va_{ijt}^B} + \frac{\tau_t [\eta e_{ij,t-1} - e_{ijt}]}{va_{ijt}^B}$$

We know the first term of the RHS by (44). We now consider the second term: given (13) and (14), we have  $e_{ijt} = \frac{\beta_j}{\alpha_j} \frac{\tilde{p}_j}{\tilde{q}_t} k_{ijt}$ . Then, thanks to (43):

$$\frac{\tau_t [\eta e_{ij,t-1} - e_{ijt}]}{va_{ijt}^B} = \tau_t \frac{\eta \frac{\beta_j}{\alpha_j} \frac{\tilde{p}_j}{\tilde{q}_{t-1}} k_{ij,t-1} - \frac{\beta_j}{\alpha_j} \frac{\tilde{p}_j}{\tilde{q}_t} k_{ijt}}{[1 - \beta_j] \frac{\tilde{p}_j k_{ijt}^B}{\alpha_j}} = \frac{\beta_j}{1 - \beta_j} \frac{\tau_t}{\tilde{q}_t} \left[ \eta \frac{\tilde{q}_t}{\tilde{q}_{t-1}} \frac{k_{ij,t-1}}{k_{ijt}} - 1 \right] \frac{k_{ijt}}{k_{ijt}^B}$$

This result with (44) imply that:

$$\begin{aligned} \frac{r_{ijt}}{r_{ijt}^B} &= \frac{1}{1 - \beta_j} \left[ 1 - \beta_j \frac{q_t}{\tilde{q}_t} + \beta_j \frac{\tau_t}{\tilde{q}_t} \left[ \eta \frac{\tilde{q}_t}{\tilde{q}_{t-1}} \frac{k_{ij,t-1}}{k_{ijt}} - 1 \right] \right] \frac{k_{ijt}}{k_{ijt}^B} \\ &= \frac{1}{1 - \beta_j} \left[ 1 - \beta_j \left[ 1 + \eta \frac{\tau_t}{\tilde{q}_{t-1}} \frac{k_{ij,t-1}}{k_{ijt}} \right] \right] \frac{k_{ijt}}{k_{ijt}^B} \end{aligned} \quad (48)$$

In this last equality appears sector  $j$ 's growth rate characterising the EP scenario. Making use of (23), (31) and (17) (with  $\hat{p}_{jt} = p_{jt}$ ), one can write that :  $\frac{k_{ijt}}{k_{ij,t-1}} = \frac{\zeta_{jt}}{\zeta_{jt}^B} gX_{ijt} =$

$\left[ \frac{q_t}{\tilde{q}_t} \right]^{\frac{\beta_j}{1-\varphi}} gX_{ijt}$ . Then (48) becomes:

$$\frac{r_{ijt}}{r_{ijt}^B} = \frac{1}{1 - \beta_j} \left[ 1 - \beta_j \left[ 1 + \eta \frac{\tau_t}{\tilde{q}_{t-1}} \left[ \frac{\tilde{q}_t}{q_t} \right]^{\frac{\beta_j}{1-\varphi}} \frac{1}{gX_{ijt}} \right] \right] \frac{k_{ijt}}{k_{ijt}^B}$$

We now check under which conditions it is verified that:

$$\frac{r_{iat}}{r_{iat}^B} < \frac{r_{ibt}}{r_{ibt}^B}, \quad i = v, w \quad (49)$$

We have the following result:

**Proposition 4** *The inequality (49) necessarily holds after some time.*

**Proof.** We know already that  $\frac{k_{iat}}{k_{iat}^B} < \frac{k_{ibt}}{k_{ibt}^B}$ ,  $i = v, w$  (see subsection 3.3). A sufficient condition for (49) to hold is then that:

$$\frac{1}{1 - \beta_a} \left[ 1 - \beta_a \left[ 1 + \eta \frac{\tau_t}{\tilde{q}_{t-1}} \left[ \frac{\tilde{q}_t}{q_t} \right]^{\frac{\beta_a}{1-\varphi}} \frac{1}{gX_{iat}} \right] \right] \leq \frac{1}{1 - \beta_b} \left[ 1 - \beta_b \left[ 1 + \eta \frac{\tau_t}{\tilde{q}_{t-1}} \left[ \frac{\tilde{q}_t}{q_t} \right]^{\frac{\beta_b}{1-\varphi}} \frac{1}{gX_{ibt}} \right] \right] \quad (50)$$

If  $X_{iat} \approx X_{ibt}$  (which is necessarily the case after some time because these two variables tend to the same limit equal to 1), then the previous inequality holds. Indeed, the two members can then be written as the values at  $\beta_a$  and  $\beta_b$  of the function  $h_t(x) = \frac{1-xA_t(x)}{1-x}$ , where  $A_t(x) = 1 + \eta \frac{\tau_t}{\tilde{q}_{t-1}} \left[ \frac{\tilde{q}_t}{q_t} \right]^{\frac{x}{1-\varphi}} \frac{1}{g}$ . Now  $h'_t(x) = \frac{1-A_t-xA'_t[1-x]}{1-x} < 0$ , because  $0 < x < 1$ ,  $A_t > 1$  and  $A'_t > 0$  (given that  $\tilde{q}_t > q_t$ ). Then the inequality follows from assumption (33).  $\square$

Under the anticipated emission-based grand-fathering sharing rule (with  $\lambda_j = \lambda$  ( $j = a, b$ )), one obtains:

$$\frac{r_{ijt}}{r_{ijt}^B} = \frac{1}{1 - \beta_j} \left[ 1 - \beta_j \left[ \frac{q_t + \tau_t}{\tilde{q}_t} + \lambda \frac{\tau_t}{\tilde{q}_{t-1}} \left[ \frac{\tilde{q}_t}{q_t} \right]^{\frac{\beta_j}{1-\varphi}} \frac{1}{gX_{ijt}} \right] \right] \frac{k_{ijt}}{k_{ijt}^B}$$

Then the above proposition follows again by a similar reasoning. It appears that the fact that firms anticipate the endowment effect or not (as explained in subsection 3.1) does not modify the previous proposition.

It is interesting to observe that the above proposition will be verified in the context of a loose EP with an increasing scarcity of fossil energy resources. Then  $q_t$  increases quicker than  $\tau_t$ , and the two members of inequality (50) tend to 1. Thus  $\frac{r_{ijt}}{r_{ijt}^B} \rightarrow \frac{k_{ijt}}{k_{ijt}^B}$ . Then (49) follows from (39).

### 3.7 Impact on the regional product

The regional product is defined by the sum of the added value of the two sectors of the region :  $VA_{it} = va_{iat} + va_{ibt}$  ( $i = v, w$ ;  $t \geq 1$ ). The impact of the EP is measured by the following ratio:

$$\frac{VA_{it}}{VA_{it}^B} = \frac{va_{iat}}{va_{iat}^B} \frac{va_{iat}^B}{VA_{it}^B} + \frac{va_{ibt}}{va_{ibt}^B} \left[ 1 - \frac{va_{iat}^B}{VA_{it}^B} \right]$$

where the share of sector  $a$  in region  $i$ 's regional product at time  $t$  at the baseline ( $va_{iat}^B/VA_{it}^B$ ) writes, given (42):

$$\frac{va_{iat}^B}{VA_{it}^B} = \frac{1}{1 + \frac{1-\beta_b}{1-\beta_a} \frac{\tilde{p}_b}{\tilde{p}_a} \frac{k_{ibt}^B}{k_{iat}^B}} \quad (51)$$

This share is thus positively related to the specialization index of region  $i$  along the baseline :  $\chi_{it}^B = k_{iat}^B/k_{ibt}^B$ .

In subsection 3.5, we have checked that  $\frac{va_{ijt}}{va_{ijt}^B}$  does not depend on  $i$  and that the inequality  $\frac{va_{iat}}{va_{iat}^B} < \frac{va_{ibt}}{va_{ibt}^B}$  is likely to be verified. Then, a sufficient condition ensuring that

$$\frac{VA_{wt}}{VA_{wt}^B} < \frac{VA_{vt}}{VA_{vt}^B} \quad (52)$$

is that  $\frac{va_{wat}}{VA_{wt}^B} \geq \frac{va_{vat}}{VA_{vt}^B}$ , or given (51), that  $\frac{k_{wbt}}{k_{wat}^B} \geq \frac{k_{vbt}}{k_{vat}^B}$ , which is equivalent to  $\sigma_t \geq 1$ <sup>7</sup>. There should not be any inversion of specialization at date  $t$  w.r.t. date 0. Now Proposition 2 states the conditions for a specialization inversion. Thus it is sufficient to inverse conditions (29) and (30) in order to guaranty that inequality (52) is verified:

(a)  $k_{wb0} - k_{vb0} > 0$  : it follows that  $\sigma_t > 1$  iff:

$$\frac{[k_{wb0} + k_{vb0}][k_{wa0} - k_{va0}]}{[k_{wa0} + k_{va0}][k_{wb0} - k_{vb0}]} > \prod_{m=1}^t \frac{\frac{\theta_{bm}}{2-\theta_{bm}}}{\frac{\theta_{am}}{2-\theta_{am}}}$$

From assumption (34) it follows that  $k_{wa0} > k_{va0}$ . The sequence  $\{\theta_{at}, t \geq 1\}$  must be in "average" high enough w.r.t. the sequence  $\{\theta_{bt}, t \geq 1\}$ . This implies that in the long term, i.e. when the sequences are close to their limit values  $\bar{\theta}_j$  ( $j = a, b$ ), the above inequality implies that the ratio  $\bar{\theta}_b/\bar{\theta}_a$  is low enough.

(b)  $k_{wb0} - k_{vb0} < 0$  : it follows that  $\sigma_t > 1$  iff:

$$\frac{[k_{wb0} + k_{vb0}][k_{wa0} - k_{va0}]}{[k_{wa0} + k_{va0}][k_{wb0} - k_{vb0}]} < \prod_{m=1}^t \frac{\frac{\theta_{bm}}{2-\theta_{bm}}}{\frac{\theta_{am}}{2-\theta_{am}}}$$

If  $k_{wa0} - k_{va0} < 0$ , then the sequence  $\{\theta_{at}, t \geq 1\}$  must be in "average" low enough w.r.t. the sequence  $\{\theta_{bt}, t \geq 1\}$ . This implies that in the long term, i.e. when the sequences are close to their limit values  $\bar{\theta}_j$  ( $j = a, b$ ), the above inequality implies that the ratio  $\bar{\theta}_b/\bar{\theta}_a$  is high enough. If  $k_{wa0} - k_{va0} > 0$ , the regions have opposite specializations (i.e.  $k_{wa0} > k_{va0}$  and  $k_{wb0} < k_{vb0}$ ), then a remarkable fact is that the above inequality is necessarily satisfied whatever the spillover parameters.

In summary, the EP affects more the regional product of Region  $w$  at date  $t$  if this region is more specialized in the energy intensive sector 1 at date  $t$ . Regarding the Belgian situation, one has  $k_{wb0} < k_{vb0}$  (where  $w$  is Wallonia and  $v$  is Flanders), so that case (b) above applies. Because it is generally expected that the technological spillovers are higher in the capital good sector ( $1 - \theta_{at} \geq 1 - \theta_{bt} \implies \theta_{at} \leq \theta_{bt}, t \geq 1$ ), one has an indication that there will be no inversion of specialization (at least after some time), so that Wallonia is likely to be more affected than Flanders by an EP such as studied in this paper.

### 3.8 Impact on the regional revenues with transfers

The total revenue (i.e. after transfers) of region  $i$  ( $i = v, w$ ) at time  $t$  writes:

$$R_{it} = r_{iat} + r_{ibt}, \quad i = v, w$$

The impact of the EP on the total revenue of region  $i$  can be measured by the ratio:

$$\frac{R_{it}}{R_{it}^B} = \frac{r_{iat}}{r_{iat}^B} \frac{va_{iat}^B}{VA_{it}^B} + \frac{r_{ibt}}{r_{ibt}^B} \left[ 1 - \frac{va_{iat}^B}{VA_{it}^B} \right]$$

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<sup>7</sup>Remember that the spread of specialization index is not modified by the environmental policy.

where we have made use of the fact that along the baseline,  $r_{ijt}^B = va_{ijt}^B$  ( $j = a, b$ )  $\Rightarrow R_{it}^B = VA_{it}^B$ . We want to check if the inequality

$$\frac{R_{wt}}{R_{wt}^B} < \frac{R_{vt}}{R_{vt}^B} \quad (53)$$

is satisfied. In the previous subsection, we have checked the conditions ensuring that the inequality  $\frac{r_{iat}}{r_{iat}^B} < \frac{r_{ibt}}{r_{ibt}^B}$  is verified. Then, a sufficient condition ensuring (53) is that  $\frac{va_{wat}^B}{VA_{wt}^B} \geq \frac{va_{vat}^B}{VA_{vt}^B}$ , or given (51), that  $\frac{k_{wbt}^B}{k_{wat}^B} \geq \frac{k_{vbt}^B}{k_{vat}^B} \Leftrightarrow \sigma_t \geq 1$ , which is the same condition that has been studied in the previous subsection.

## 4 Conclusion

The model presented in this paper has been designed to study the burden sharing of pollution sharing in the context of a Kyoto-like protocol in a multi-regional multi-sectoral economy. In order to get the dynamic picture, we have considered time-dependent intra-sectoral spillovers across regions and learning-by-investing in each sector. In such a context, we have shown progressively how the typical wisdom gathered for the static case can be altered. In particular, we have disentangled the main price-based and quantity-based mechanisms which determine the impact of environmental policy at different levels (sectoral value-added and regional notably).

Within this framework, we have been able to extract some qualitative predictions for a country like Belgium. Nonetheless, a much more serious quantitative assessment is needed, and this would require a rigorous calibration of the model, including the exogenous price processes involved. A major difficulty comes from the fact that some processes like the price of pollution permits are not very well known given the short historical record. Alternatively, some reasonable scenarios could be considered. We are currently working in this line of research.

## 5 References

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## 6 Appendix

### 6.1 Proof of Proposition 1

The system (18-19) can be rewritten in a planar stacked form :

$$\mathbf{K}_{jt} = \zeta_{jt} \rho_t \mathbf{A}_{jt} \mathbf{K}_{j,t-1} \quad (54)$$

where

$$\mathbf{A}_{jt} = \begin{bmatrix} 1 & 1 - \theta_{jt} \\ 1 - \theta_{jt} & 1 \end{bmatrix}$$

and

$$\mathbf{K}_{jt} = \begin{bmatrix} k_{wjt} \\ k_{vjt} \end{bmatrix}$$

The eigenvalues of  $\mathbf{A}_{jt}$  are  $\theta_{jt}$  and  $2 - \theta_{jt}$ . Denote  $\mathbf{P} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$  and

$\mathbf{D}_{jm} = \zeta_{jm} \rho_m \begin{bmatrix} 2 - \theta_{jm} & 0 \\ 0 & \theta_{jm} \end{bmatrix}$ . It is easy to check that the vector  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  is an eigenvector of  $\mathbf{A}_{jt}$  associated with the eigenvalue  $2 - \theta_{jt}$ ,  $\forall t \geq 1$ . The same can be said about the vector  $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$  and the eigenvalue  $\theta_{jt}$ . Therefore the eigenvectors are time independent. It follows that

$$\mathbf{K}_{jt} = \zeta_{jt} \rho_t \mathbf{P} \begin{bmatrix} 2 - \theta_{jt} & 0 \\ 0 & \theta_{jt} \end{bmatrix} \mathbf{P}^{-1} \mathbf{K}_{j,t-1}$$

Then elementary backward successive substitutions leads to the solution to the system (54) :

$$\mathbf{K}_{jt} = \mathbf{P} \left[ \prod_{m=1}^t \mathbf{D}_{jm} \right] \mathbf{P}^{-1} \mathbf{K}_{j0} \quad (55)$$

Then noticing that  $\mathbf{P}^{-1} = \mathbf{P}/2$ , one obtains (20) and (21).

### 6.2 Proof of Proposition 3

The comparison (46) amounts to study the  $x$ -function :  $h_t(x) = \frac{1-A_t x}{1-x} B_t^{\frac{x}{1-\varphi}}$ , where  $0 \leq$

$x < 1$ ,  $A_t = \frac{1}{1 + \frac{\tau_t - \tau_{t+1} \frac{\lambda}{1+r}}{q_t}}$ ,  $B_t = \prod_{m=1}^t \left[ \frac{1}{1 + \frac{\tau_m - \tau_{m+1} \frac{\lambda}{1+r}}{q_m}} \right]$  and  $0 < B_t < A_t < 1$ . Notice

that :  $h'_t(x) = e^{\frac{x}{1-\varphi} \ln(B_t)} \left[ \frac{1-A_t x}{1-x} \frac{\ln(B_t)}{1-\varphi} + \frac{1-A_t}{[1-x]^2} \right]$ . Therefore, the sign of  $h'_t(x)$  is the sign of the polynomial of degree 2 :  $P_t(x) = [1-x][1-A_t x] \frac{\ln(B_t)}{1-\varphi} + 1 - A_t = \frac{A_t \ln(B_t)}{1-\varphi} x^2 - \frac{\ln(B_t)}{1-\varphi} [1+A_t] x + \left[ 1 - A_t + \frac{\ln(B_t)}{1-\varphi} \right]$ . The discriminant is :

$\Delta = \left[ \frac{\ln(B_t)}{1-\varphi} [1+A_t] \right]^2 - 4 \frac{A_t \ln(B_t)}{1-\varphi} \left[ 1 - A_t + \frac{\ln(B_t)}{1-\varphi} \right] = \left[ \frac{\ln(B_t)}{1-\varphi} [1-A_t] \right]^2 - 4 \frac{A_t [1-A_t] \ln(B_t)}{1-\varphi} > 0$ . Thus  $P_t(x)$  admits two real roots. These roots are :

$$\begin{aligned} x_{1t} &= \frac{1 + A_t - [1 - A_t] \sqrt{1 - \frac{4A_t}{1-A_t} \frac{1-\varphi}{\ln B_t}}}{2A_t} \\ x_{2t} &= \frac{1 + A_t + [1 - A_t] \sqrt{1 - \frac{4A_t}{1-A_t} \frac{1-\varphi}{\ln B_t}}}{2A_t} \end{aligned}$$

with  $x_{2t}$  is higher than 1, and is thus of no interest because we limit the analysis of  $h_t(x)$  for  $0 \leq x < 1$ .  $x_{1t}$  is at its maximum value (equal to 1) when  $1 = \frac{4A_t}{1-A_t} \frac{1-\varphi}{\ln B_t}$ . Thus  $x_{1t} \leq 1$ .  $P_t(x)$  is concave and is thus negative when  $x \leq x_{1t}$  and positive when  $x_{1t} \leq x$ , which implies that  $h_t(x)$  is decreasing when  $x \leq x_{1t}$  and increasing when  $x_{1t} \leq x$ . (46) can only be verified if  $h_t(x)$  is decreasing at least on some part of the interval  $[0, 1[$ , which supposes that  $x_{1t} > 0$ . A sufficient condition for (46) to be satisfied is then that  $\beta_b < \beta_a < x_{1t}$ , which is more likely to be verified the more  $x_{1t}$  is close to 1, that is when  $B_t$  is small enough. In practice, it is not necessary that  $x_{1t}$  should be close to 1, because  $\beta_j$  ( $j = a, b$ ) can be interpreted as the share of fossil energy in production, and should thus be closer to 0 than to 1.